Global Positioning System

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1 Introduction

GPS is a satellite-based system for positioning and navigation with coverage along the whole globe. This coverage is possible thanks to 24 MEO satellites, although as of August 2006 the constellation is formed by 29 satellites. The applications that can make use of this technology are increasing even nowadays (specially the non-militar ones), despite the fact that it is nearly 30 years old. The current uses of the GPS system includes military uses, navigation, applied in cars, ships and air-planes; mobile satellite communications, location-based services and games, and also as a precise time reference (every GPS satellite has is own atomic based clock).

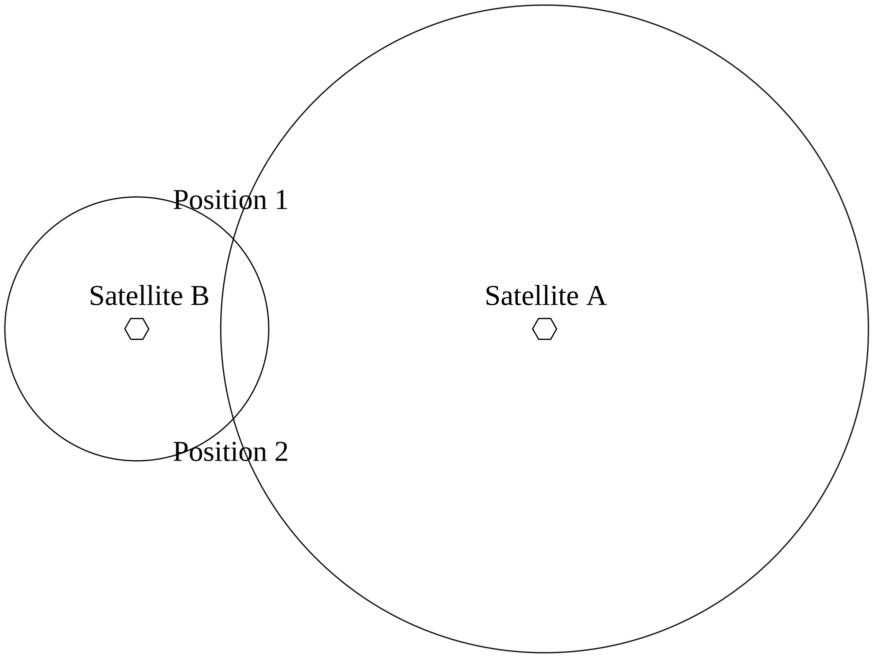
Nevertheless, the original idea of this technology came in war time with similar ground-based radio navigation systems, such as LORAN developed in the early 1940s, and used during World War II. After some tests with similar satellite-based technologies the first GPS satellite was launched in February 1978. In 1983 Ronald Reagan announced the civilian use of the GPS. It was in December 1993 when the GPS system achieved a fully working infrastructure. Only one month later, the system was running with 24 satellites.

In this report we're trying to make a short overview over this system. In the first part we will take a look to the physical concepts that allows to position every GPS user. Then we'll take a look of the infrastructure that makes this possible, and we will finish with an analysis on the GPS signal.

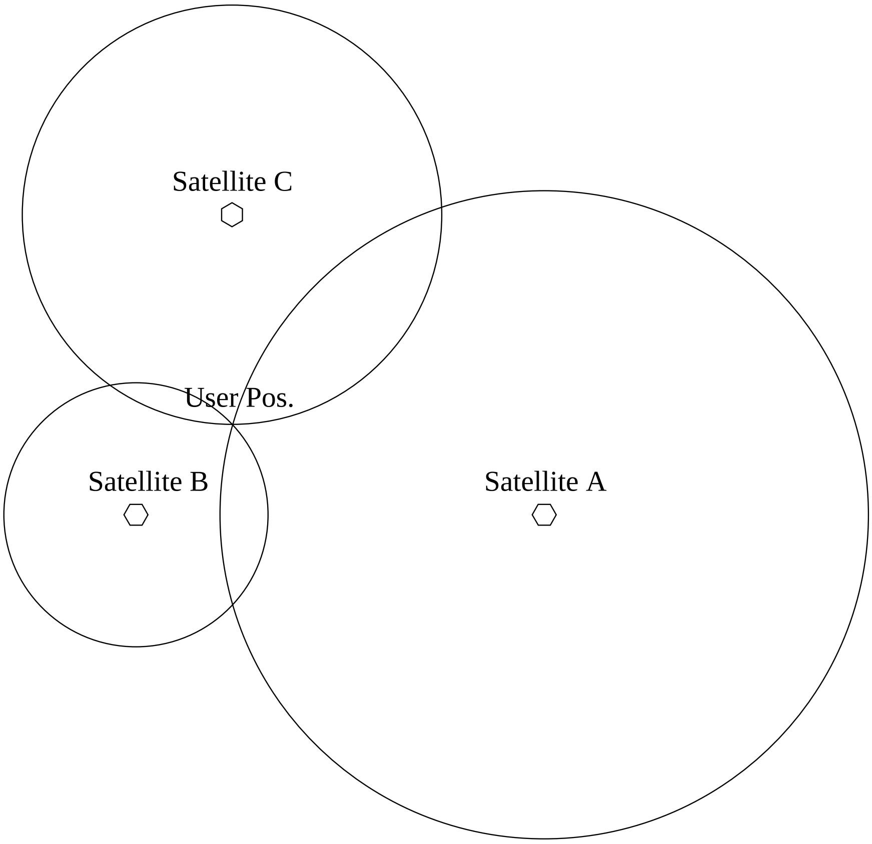
2 Navigation Principles

2.1 Basic Positioning

The GPS system bases its properties in the concept of TOA (Time of arrival). If we suppose that we know exactly the position of a certain amount of satellites and the travel time of a signal from the satellite to our receiver we are able to know exactly our position. The easiest example can be done on a 2D plane with 2 crossing circles. If we suppose that in the centre of the circle we have the GPS satellite, and in the circumference we have the reach of a broadcast satellite signal; we can establish two possible positions for the GPS receiver, one in every point where the circumferences are crossing. If we know the time when the satellite sent the message, and the time when the message was received, we get the distance between the user and the satellite simply by multiplying the total time per the propagation speed.



Anyway, in this case, we don't know in which crossing point is the user. We can tell this if we consider that the satellites will be always in a upper plane related to the user position. The other option is to employ a third satellite, that will show us which is our crossing point or true position, and could also give us some extra information for error correction.



2.2 GPS Positioning: Troubles and Solutions

All this gets more complicated when we talk about the 3D world, but in the end, what we need is to add an extra satellite. With three satellites we get two points, as we did with two satellites in our previous 2D world. The solutions to choose the right user position are also the same. The solution that GPS system takes is to have 4 satellites always available for every user on the earth. As a result, the system needs at least 24 satellites running for global coverage.

2.2.1 The Reference Coordinate System

But there is another problem when we face with our real world. The reference coordinate system, in which we have to take care of the ”almost” spherical surface of the earth. Besides, instead of considering a static coordinate system, for GPS is easier to use a dynamic one, that rotates with the earth. With all this considerations, we use a Earth-Centred Earth-Fixed (ECEF), so we consider the earth as a fixed object in the reference system; and we will take as a earth model the World Geodetic System 1984 (WGS-84), that uses a ellipsoidal model of the earth's shape, and take care of the gravitational irregularities. All this data is supplied to the GPS receiver through the ephemeris information for a precise calculus of the right position of the user.

2.2.2 Dealing with the non-synchronised clocks

But now we have to face another problem: the biggest source of error, a non-synchronised clock. This is not a problem for the satellites, which usually get time updates through the monitoring system and also have high-precision atomic clocks. But it is nearly impossible for a cheap GPS receiver not to have this synchrony problem. So we should assume this error t_u, and we will put that term in the position equation.

But from the start let's suppose that we have a synchronised system. We will try to estimate that error later in our calculus while we offer another service: an extremely precise time reference. First, we want to know the time-of-arrival (TOA) from the satellite to the user, so we need to know in advance what kind of signal is sending every satellite in every space of time. For this purpose the space vehicle (SV) sends a well-known code that every user station could recognise, not only the time that it was sent, but also which satellite sent it. The receiver device will start to reproduce the signals that it could expect to get from the satellite, until one signal (replica) will match with the received signal, when it will know exactly the TOA. Of course, this time includes the error that a bad synchronised clock could have introduced (t_u).

So, now we will put all the knowledge we have from the system to get our position and system time. The data that is known is the position of the satellites (x_i, y_i and z_i), that can be calculated making use of the ephemeris data; and the TOA from every SV (ρ_i), with the clock error included, that we have get through the process described before. The unknown variables are the 3 dimensional x_u, y_u and z_u variables and the t_u error variable. If we take in mind that we have always at our sight at least 4 satellites we can make this set of 4 equations, one for every SV:

$$\rho_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_y)^2} + ct_u \tag{1}$$

$$\rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_y)^2} + ct_u \qquad (2)$$

$$\rho_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_y)^2} + ct_u \qquad (3)$$

$$\rho_4 = \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_y)^2} + ct_u \qquad (4)$$

Anyway, in case that we didn't have available 4 satellites, we could left out the calculus of the altitude with an appropriate conversion of the equations.

2.2.3 Other sources of error

We also have to consider that in the real world we also suffer precission problems due to variations in the propagation speed of the light along the layers of the atmosphere or with external radio-frequency (RF) signals. All this factors should be accurately considered to get a efficient model. The first solution adopted to cut down the SPS error is to add a second signal called L2. The L2 has a different frequency that L1 has, so the delay it suffers when crossing the atmosphere is (proportionally) different to the L1 delay. This is the basis of the PPS system, and allows a predictable accuracy of at least 22m in the horizontal plane and 27,7m in the vertical plane. Besides another signal is being studied for additional ionospheric correction, but has not been introduced yet.

Of course, there are much more techniques that try to minimise even more this error without the need of another satellite signal, but they could be a topic for an entire book and are not covered in this report.

3 GPS Segments

The GPS system is composed, as far as we have seen, of (at least) 24 Satellites and a variable number of users, but we need a third element in our system, capable of monitoring the satellites. But let's see them one by one.

3.1 GPS Satellites

GPS system is comprised of a network of 24 satellites along 6 different Earth-centred orbital planes with 4 satellites in every plane. Every satellite does 2 turns around the earth per day. All the satellites orbites at a distance of (approximately) 26.600 km from the earth centre. The satellites are disposed in a manner that the users can see 4 different satellites from every point over the globe, in order to get its current position.

3.2 Operational Control System (OCS)

The responsibilities of the OCS segment covers the maintenance of the satellites. This includes periodical station keeping, monitoring the health of the satellite subsystem and updating the satellite clock, almanac and ephemeris.

For this tasks the OCS segment employs three different physical components: the remote monitor station that is always making calculations of its own position. The exact position of the monitor station is well-known, so every failure of the system is noticeable. So, all collected data is sent to the next component: the master control station (or MCS): one station which works as a central for every monitoring information, computes the corrections needed and send it through our next component: the ground antenna. The objective of this last component is to send all the information back to the satellite that will correct its position consequently. Of course, all the receiving and transmitting components are replied along the globe in order to maximise satellite coverage, although the MCS can remain centralised in one location, Falcon Air Force Base, to be exact.

3.3 User Receiving Equipment

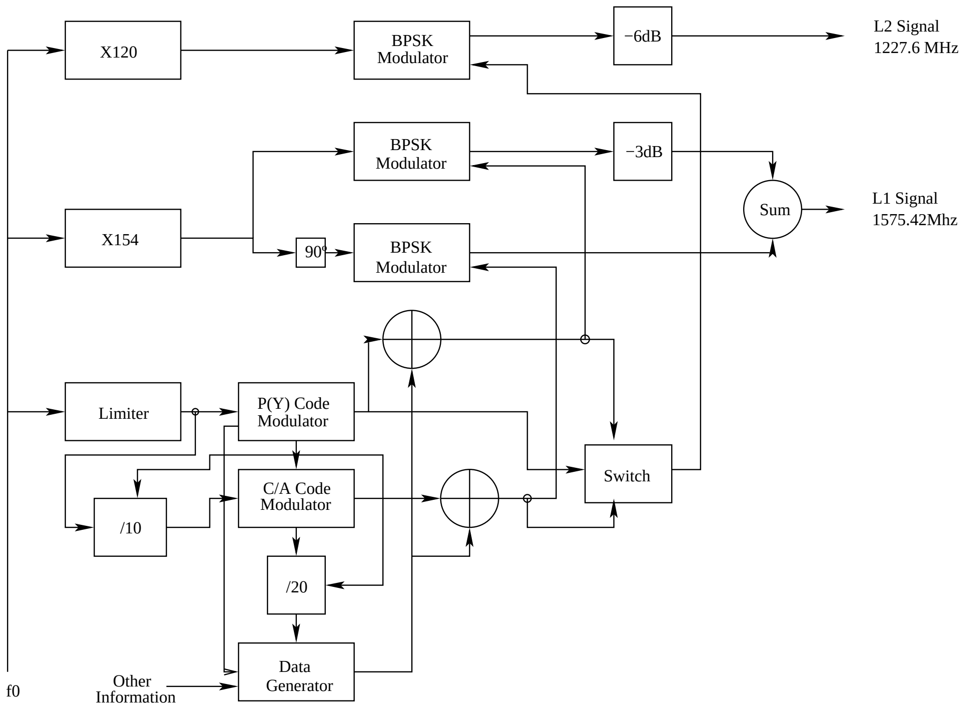
Typically referred to as a "GPS Receiver", this conforms the last segment and the one that a user should have in order to determine his position. Its basics components are a antenna connected to the receiver, one processor that interfaces with a I/O control display unit. We can make two big groups of GPS receivers: the SPS (Standard Positioning System) and PPS (Precise Positioning System), depending in the precision that the system could achieve. The PPS is primarily intended for military, while the SPS is available worldwide. The difference can be made thanks to an extra signal (L2) which sports strong cryptographic features such as anti-spoofing (AS) and selective availability (SA).

The GPS receivers have improved a lot during this years. They have become smaller, cheaper and more precise due to the advances in electronic matters (the firsts models were analogic), so nowadays is not difficult to find GPS receivers costing barely 100$.

4 GPS Signal

Two are the main carrier frequencies that a GPS satellite transmits: L1, the primary frequency; and L2 the secondary frequency, which (as we explained before) enables PPS. The first runs at 1575,42 MHz while L2 runs at 1227,6 MHz. Nevertheless, their signals do not interfere significantly each other due to the use of unique and uncorrelated PRN (pseudo-random noise) to modulate the signal. Besides, every satellite has his own PRN code, so that the signals doesn't even interfere between different satellites. This mechanism also enables the user to identify the satellite from which he is getting the signal thanks to a technique called CDMA (code division multiple access). Besides, there is a lot of data that we need to broadcast to the users, that we will modulate with these frequencies, like anomalies in the reference time, the eccentricity of the satellite orbit or the operating week.

But let's see with more detail how the satellite generates the signals with the help of the following diagram, as we can see it in the book *Understanding GPS* (refer to the bibliography for more details).



As we can see, the originating frequency that the satellite uses to generate the rest is f_0 that is formerly 10,23 MHz in the earth, but it is readjusted in the order of the mHz in the satellites due to relativistic effects. As we see, L1 and L2 signals are multiplied and modulated using a BPSK (Bi-Phase Shift Keying). L1 signal also includes the C/A code and the rest of the data. Nevertheless, L2 can have it or not, depending on the state of the switch. The P(Y) code term refers to the ciphering capability of the signal, that enables to choose the anti-spoofing in the satellite in the L2 (PPS) signal.

4.1 P Code

In fact, when we talk about the P code, we refer to a non-ciphered precision code. This code is obtained thanks to two pseudo-random noises (PRN) with the same chip rate (we avoid to talk about bit rate, because there is no data transmitted with this code), the first one has a period of 1,5 seconds and 15.345.000 chips, while the second is 37 chips longer. The reason for this is that the resulting chip periods are coprime between them, and if we start them at the same time, they will coincide only every $1,5 * 15.345.037$ that is approximately 38 weeks. Nevertheless, the GPS system resets both PRN every week, so that every GPS satellite can use different pieces of the code. Of course, the week is known through the C/A code.

4.2 Coarse (or Clear)/Acquisition code (C/A)

The most elemental unit of GPS data that a satellite broadcasts is a C/A code, that has a specified duration of 1ms with a chip rate of 1.024 MHz so the chip time is $0,977ns$. With 20 C/A codes we get a one unit of navigation data, 30 of this conforms one word. With 10 words you get a sub-frame. 5 sub-frames conforms a page, and 25 pages comprises the maximum length of the GPS data, that will be repeated and has a length of $12,5$ minutes. In fact, each bit of the GPS signal is as long as $20ms$, or one navigation data. So the resulting rate is 50 Hz, and the modulation of the C/A codes remains the same every 20 codes.

The navigation broadcasted in this structure comprises a lot of variables that the users and satellites need to operate. For example, the first word in every sub-frame is used basically to detect its beginning. The second word codifies the time of week using 17 bits with a precision of 6 seconds; 2 bits more are used for flagging operation modes on the satellite, like the anti-spoof; 3 bits more are used for identifying the sub-frame, while the rest are used for parity checks. The rest of the words contain very detailed navigation data, like the calculations of the ionospheric delay, that we will not cover in this report, but you can explore with the bibliography.

One nice feature of the C/A code is its correlation, near to orthogonal. This feature enables us to work with weak signals (with GPS we have to take care of huge attenuation, in the order of $-130dBm$), that will be impossible to detect in other case.

5 Bibliography

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